

# Input trajectory transfer in heterogeneous dynamic systems for muscular stimulation

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*Abstract: Transferring input or output trajectories between dynamic systems is a key element in multi-agent learning control or to reuse previously learned trajectories if the dynamics have changed. However, if the dynamics are unknown or non-linear, this transfer is not trivially found. We propose a method to estimate this transfer by looking at the difference in dynamics for structurally similar systems. The method is introduced and applied to the stimulation dynamics of muscles. Despite the muscles having complex, non-linear dynamics we show the method can successfully estimate the difference in dynamics and transfer an input trajectory from one muscle to the other.*

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## I. Introduction

In learning control, the transfer of input or output trajectories between systems is a crucial element. It is used to transfer learned trajectories between multiple agents during learning or to use trajectories that have been learned in the past on a different system to a new target. However, if the systems have dissimilar dynamics, a direct transfer of trajectories might not be possible and could lead to unwanted behaviour. Relearning all trajectories might not be feasible or very costly. To address this issue, various methods have been proposed, such as using adaptive control [1] [2] or finding a translation module that can transform the trajectories to be used on the other agent [3]. While these methods have shown potential, they may have limitations, such as limiting the dynamic capabilities of the system, especially when the number of agents increases. Additionally, prior works have mostly focused on transfer of output trajectories, and either assume model knowledge [3] or that the trajectories are well-below the system's bandwidths.

To overcome these limitations, we propose a method for identification of dynamic model differences for structurally similar models by exciting both systems with a short test input or reusing data from previous learning episodes. This avoids a full system identification of all involved systems, which can be very difficult or impossible. Using these model differences, we derive a model for a dynamic input transfer that enables the transfer of input trajectories between dynamic systems without knowledge of the possibly complex individual dynamics.

We demonstrate the feasibility of our method using simplified stimulation dynamics of muscles, which are highly non-linear, time-variant, and complex. The dynamics of different muscle groups, patients, or even from one day to another can differ greatly, making a direct application of previously learned input trajectories

infeasible. Furthermore, a full system identification for each muscle or re-learning all trajectories can be very costly or not possible. Our method can achieve similar stimulation output with a given stimulation input despite model differences and without full system identification.

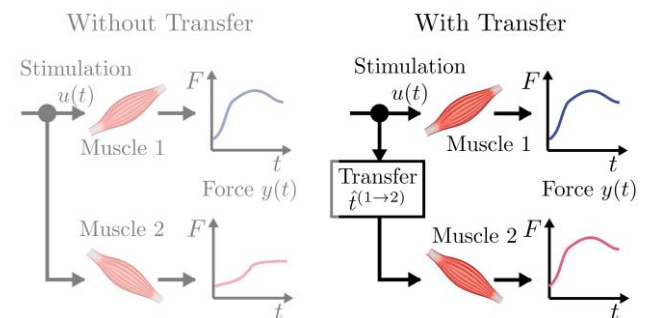


Figure 1: Use of input trajectory  $u(t)$  in dissimilar systems vs. the use of a transfer module. The transfer module  $\hat{t}^{(1 \rightarrow 2)}$  transforms the input  $u(t)$  to yield the same output  $y(t)$  in both systems.

## II. Method

Consider two dynamic systems  $f^{(1)}: \mathbb{R}^N \rightarrow \mathbb{R}^N$  and  $f^{(2)}: \mathbb{R}^N \rightarrow \mathbb{R}^N$  describing the input-output behavior of two agents with

$$\mathbf{y}^{(i)} = f^{(i)}(\mathbf{u}),$$

with  $\mathbf{u} \in \mathbb{R}^N$  being the input trajectory of length  $N$  and  $\mathbf{y}^{(i)} \in \mathbb{R}^N$  being the corresponding output trajectory of same length of system  $i$ . For any given non-trivial  $\mathbf{u} \in \mathbb{R}^N$  in general  $f^{(1)}(\mathbf{u}) \neq f^{(2)}(\mathbf{u})$ .

Let  $f^{(1 \rightarrow 2)}: \mathbb{R}^N \rightarrow \mathbb{R}^N$  be the deviation dynamics of  $f^{(1)}$  and  $f^{(2)}$  with

$$f^{(2)}(\mathbf{u}) = f^{(1 \rightarrow 2)}\left(f^{(1)}(\mathbf{u})\right) \forall \mathbf{u}.$$

Correspondingly, let  $t^{(1 \rightarrow 2)}: \mathbb{R}^N \rightarrow \mathbb{R}^N$  be the *ideal input transfer* of  $f^{(1)}$  and  $f^{(2)}$  with

$$f^{(1)}(\mathbf{u}) = f^{(2)}\left(t^{(1 \rightarrow 2)}(\mathbf{u})\right) \quad \forall \mathbf{u},$$

with  $t^{(1 \rightarrow 2)}(\mathbf{u})$  being the transformed input, that yields the same output on system  $f^{(2)}$ . If  $f^{(1)}$ ,  $f^{(2)}$  and  $f^{(1 \rightarrow 2)}$  are permutable functions (cite), then in the ideal case

$$t^{(1 \rightarrow 2)} = f^{(2 \rightarrow 1)}.$$

In practice, we do not assume  $f^{(1)}$  and  $f^{(2)}$  to be known, also the assumption of permutability of the systems' dynamics might not be fulfilled perfectly. Let  $\hat{f}^{(1 \rightarrow 2)}$  approximate  $f^{(1 \rightarrow 2)}$ . Similarly, let  $\hat{t}^{(1 \rightarrow 2)}$  approximate  $t^{(1 \rightarrow 2)}$  and we assume that

$$\hat{t}^{(1 \rightarrow 2)} = \hat{f}^{(2 \rightarrow 1)}.$$

We do not assume to know the order of  $f^{(1)}$  and  $f^{(2)}$ , however we assume that both dynamics are structurally similar in a way that the model difference  $\hat{f}^{(1 \rightarrow 2)}$  can be approximated by a biproper transfer function of a fixed degree  $K$  with

$$\hat{f}^{(1 \rightarrow 2)} = \hat{F}^{(1 \rightarrow 2)}(s) = \alpha \frac{\prod_{k=1}^K (s - a_k)}{\prod_{k=1}^K (s - b_k)},$$

with  $\alpha \in \mathbb{R}, \forall k a_k, b_k \in \mathbb{C}$ .

The model  $\hat{f}^{(1 \rightarrow 2)}$  can be obtained by optimization using the outputs generated from applying a common test input to both systems or using data from previous learning episodes.

### III. Simulation Example

As an example, to demonstrate the input transfer estimation in simulation, we use the Hill Muscle Model as shown in Fig. 2, which models a muscles force response  $F$  to a stimulation signal  $u$  [4].

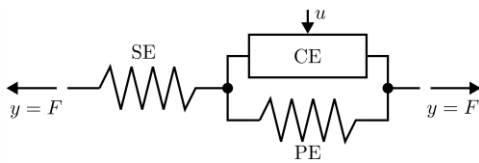


Figure 2: Hill Muscle Model containing a serial element (SE), a parallel element (PE) and a contractile element (CE).

In the example we use two different muscles, with differences in their activation dynamics and model parameters to model different output behavior to the same stimulus. The model is non-linear and complex; however, we do not model any real-world effects such as noise, hystereses or time-variant effects. This is only a feasibility study and proof-of-concept to show that the proposed model can work for even highly non-linear dynamics. To obtain an estimate of the deviation dynamics  $\hat{f}^{(1 \rightarrow 2)}$ , we stimulate both muscles with a sine-sweep of 4 seconds as shown in Figure 3. From that we estimate a model  $\hat{f}^{(1 \rightarrow 2)}$  with an order of  $K = 2$ . The model is then used to determine  $\hat{t}^{(1 \rightarrow 2)}$  to transfer an input trajectory  $u(t)$  from system  $f^{(1)}$  to system  $f^{(2)}$  as shown in Fig. 4.

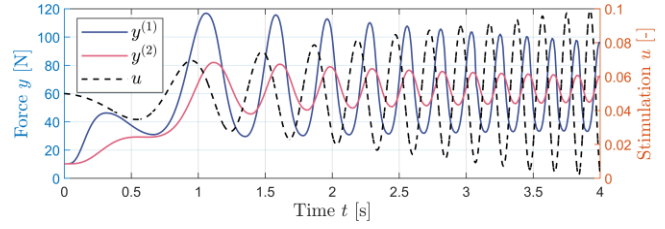


Figure 3: Stimulation signal  $u(t)$  and both systems' responses  $y^{(1)}$ ,  $y^{(2)}$  for the estimation of the deviation dynamics  $f^{(1 \rightarrow 2)}$

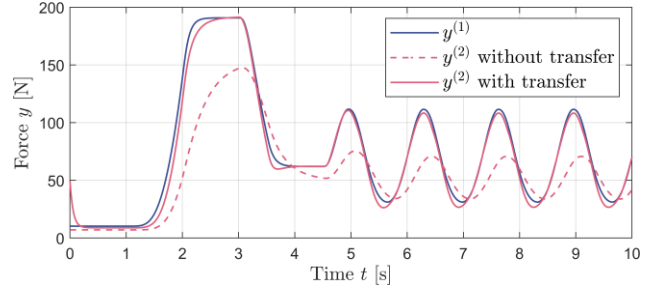


Figure 4: Output signals of both systems to the same input  $u(t)$  with two steps and a sinusoidal stimulation. The output of the target system  $f^{(2)}$  is shown with and without using the estimated input transfer  $\hat{t}^{(1 \rightarrow 2)}$ .

Using the transformed input yields an output  $y^{(2)}$  close to the target output  $y^{(1)}$  with a significant improvement over the non-transformed input.

### IV. Conclusion and Outlook

The proposed method enables input transfer between heterogenous, unknown dynamics by estimating the difference in dynamics. This allows for reusing previously learned trajectories on a source system to a new target system by only estimating the less complex model difference between both systems. This can contribute to improved cooperative learning even in heterogeneous groups or the transfer of trajectories from simulations to real-world systems in biomedical applications.

#### ACKNOWLEDGMENTS

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